PHYSICAL REVIEW E **73**, 025701(R) (2006)

# Persistence and the random bond Ising model in two dimensions

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(Received 14 November 2005; published 3 February 2006)

We study the zero-temperature persistence phenomenon in the random bond $\pm J$ Ising model on a square lattice via extensive numerical simulations. We find strong evidence for "blocking" regardless of the amount disorder present in the system. The fraction of spins which *never* flips displays interesting nonmonotonic, double-humped behavior as the concentration of ferromagnetic bonds $p$ is varied from zero to one. The peak is identified with the onset of the zero-temperature spin glass transition in the model. The residual persistence is found to decay algebraically and the persistence exponent $\theta(p) \approx 0.9$ over the range $0.1 \leqslant p \leqslant 0.9$. Our results are completely consistent with the result of Gandolfi, Newman, and Stein for infinite systems that this model has "mixed" behavior, namely positive fractions of spins that flip finitely and infinitely often, respectively. [Gandolfi, Newman and Stein, Commun. Math. Phys. **214**, 373 (2000).]

DOI: 10.1103/PhysRevE.73.025701 PACS number(s): 05.20.−y, 05.50.+q, 05.70.Ln, 64.60.Cn

## I. INTRODUCTION

In recent years there has been considerable interest in the "persistence" problem [1–11] and it has been studied theoretically in a wide range of systems. Generically, this problem is concerned with the fraction of space which persists in its initial ($t=0$) state up to some later time. Thus, when studying the nonequilibrium dynamical behavior of spin systems at zero-temperature we are interested in the fraction of spins, $P(t)$, that persists for $t>0$ in the same state as at $t=0$.

It has now been established for quite some time that for the pure ferromagnetic two-dimensional Ising model, $P(t)$ decays algebraically [1–4]

$$P(t) \sim t^{-\theta}, \tag{1}$$

where $\theta=0.209\pm0.002$ [5]. Similar algebraic decay has been found in numerous other systems displaying persistence [10,11]. However, computer simulations of the Ising model in high dimensions [3], $d>4$, and the $q$-state Potts model [12] ($q>4$) have suggested the presence of a nonvanishing persistence probability as $t\to\infty$; this feature is sometimes referred to as "blocking" and has also been found to be present in some models containing disorder [5,6,13–15]. Clearly, if $P(\infty)>0$, the problem can be reformulated by restricting attention only to those spins that eventually do flip. Therefore, we can study the behavior of the residual persistence

$$r(t) = P(t) - P(\infty). \tag{2}$$

Most of the initial effort was restricted to studying pure systems and, it's only fairly recently that the persistence behavior of systems containing disorder has been studied [5,6,13–15]. Very recently [16], the local persistence exponent for the axial next-nearest neighbor Ising model has been

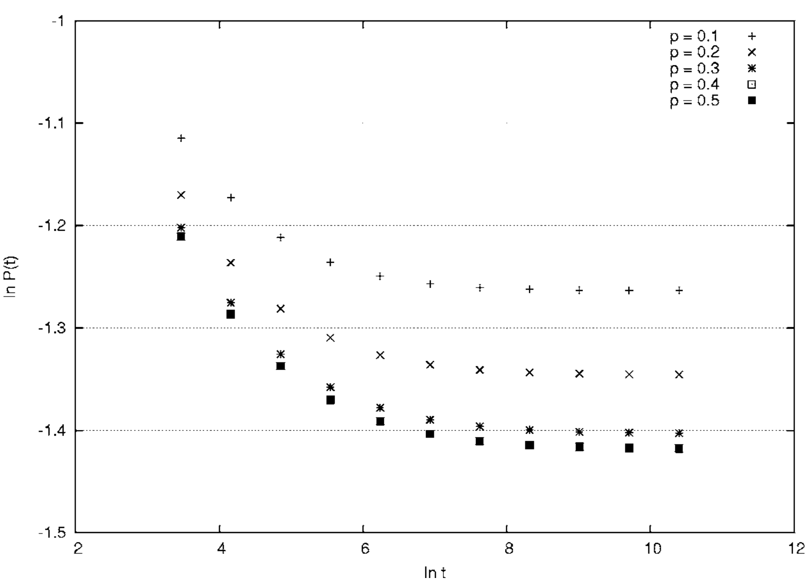


FIG. 1. A log-log plot of the persistence against time for a range of bond concentrations, $0.1 \leqslant p \leqslant 0.5$. Note that the data for $p=0.5$ are superimposed over those for $p=0.4$.

 

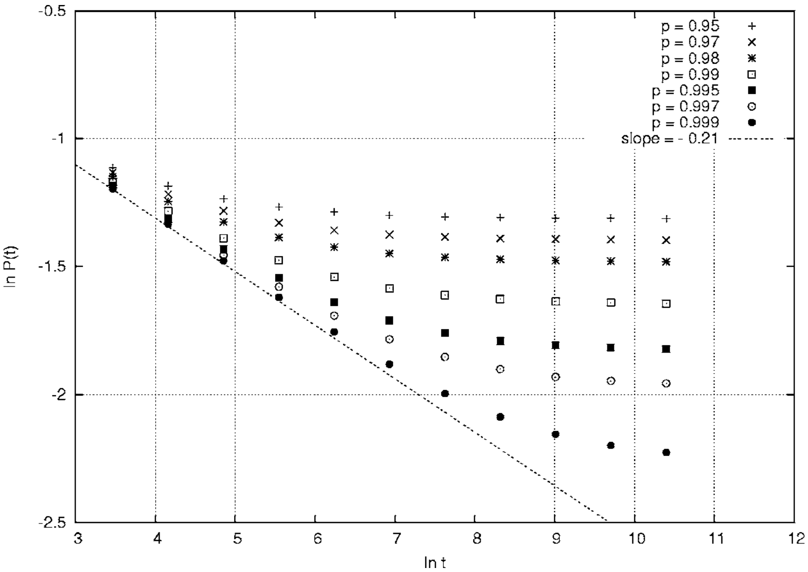


FIG. 2. A plot of $\ln P(t)$ against $\ln t$ for $0.95 \leqslant p \leqslant 0.999$. The straight line, corresponding to the behavior for the pure ($p=1.0$) case, has gradient −0.21.

estimated to be $\theta=0.69\pm0.01$; a value considerably different to that found for the ferromagnetic Ising model.

Numerical simulations of the bond diluted Ising model [5,6] indicate that the long-time behavior of the system depends on the amount of disorder present. For the weakly diluted system [5], there is evidence of nonalgebraic decay prior to blocking. For the strongly diluted model, on the other hand, the residual persistence probability decreases exponentially for large times [6].

Although the presence of a blocked state has also been suggested [13–15] for the random bond Ising model in two dimensions, the long-time behavior of the residual persistence has not been investigated to date and is still an open question.

In this work we attempt to fill the gap by presenting new results of extensive computer simulations of the two-dimensional random bond Ising model on a square lattice for a wide range of bond concentrations. Our main objective is to investigate the persistence behavior as a function of the ferromagnetic bond concentration. As we shall see, we find strong evidence for blocking regardless of the amount of disorder present in the system. Furthermore, unlike the bond-diluted case [5,6], here the qualitative behavior of the model does not appear to depend on the concentration of the disorder.

In Sec. II we introduce the model and give brief details about the method used to perform the simulations. In Sec. III we discuss the results and finish with some concluding remarks.

## II. THE MODEL

The Hamiltonian for our model is given by

$$H=-\sum_{\langle ij\rangle} J_{ij}S_iS_j, \tag{3}$$

where $S_i=\pm 1$ are Ising spins situated on every site of a square lattice with periodic boundary conditions and the quenched ferromagnetic exchange interactions are selected from a binary distribution given by

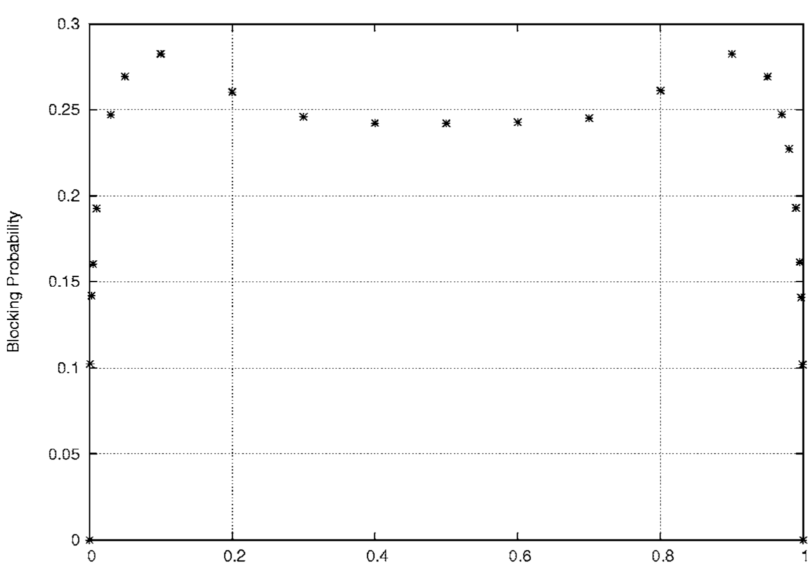


FIG. 3. A plot of the blocking probability, $P(\infty)$, against the bond concentration, $p$.

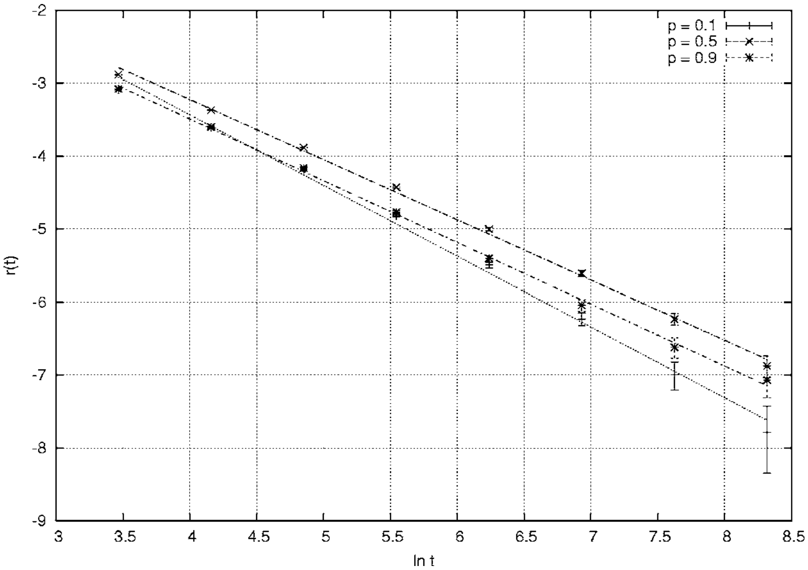


FIG. 4. Here we show a log-log plot of the residual persistence against time for selected bond concentrations. The straight lines are guides to the eye and indicate that $\theta$=0.97(8), 0.83(3), and 0.85(4) for $p$=0.1, 0.5, and 0.9, respectively.

$$P(J_{ij}) = (1-p)\,\delta(J_{ij}+J) + p\,\delta(J_{ij}-J), \tag{4}$$

where $p$ is the concentration of ferromagnetic bonds and we set $J=1$; the summation in Eq. (3) runs over all nearest-neighbor pairs only. Note that for $p=1/2$ and $p=1$ we obtain the Ising spin glass and the pure ferromagnetic Ising models, respectively.

Initial runs for a range of ferromagnetic bond concentrations were performed for lattices of linear dimensions ranging from $L=250$ to $L=1000$. No appreciable finite-size effects were evident for the range of values considered. As a result, the data presented in this work were obtained for a lattice with dimensions $500 \times 500$ (=$N$).

Each simulation run begins at $t=0$ with a random starting configuration of the spins and then we update the lattice via single spin flip zero-temperature Glauber dynamics [5]. The updating rule we use is always flip if the energy change is negative, never flip if the energy change is positive, and flip at random if the energy change is zero.

The number, $n(t)$, of spins which have never flipped until time $t$ is then counted. The persistence probability is defined by [1]

$$P(t) = [\langle n(t)\rangle]/N, \tag{5}$$

where $\langle\cdots\rangle$ indicates an average over different initial conditions and $[\cdots]$ denotes an average over samples. Averages over at least 100 different initial conditions and samples were performed for each run undertaken.

## III. RESULTS

We now discuss our results. The behavior of the persistence probability is displayed in the log-log plot shown in Fig. 1 ($0.1 \leqslant p \leqslant 0.5$). The problem is symmetric about the spin-glass ($p=0.5$) case and, as a check on the numerics, we confirmed that similar plots were obtained for ($0.5 \leqslant p \leqslant 0.9$). Note that the error bars are smaller than the

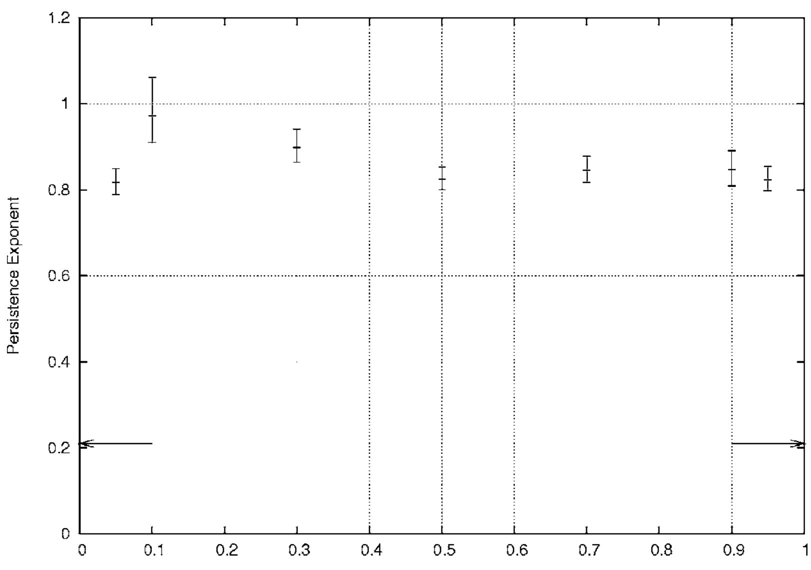


FIG. 5. A plot of $\theta(p)$ against $p$. For reference, the arrows indicate where the pure, ferromagnetic ($p$=1.0) and antiferromagnetic ($p$=0), values appear on the plot.

size of the data points. It is clear from the figure that $P(t)$ is finite in the long-time limit. Hence, the system is blocked. The blocking probability depends on $p$, the concentration of ferromagnetic bonds present. However, it would appear that blocking occurs for all of the values of $p$ considered.

To explore the blocking feature further, we plot in Fig. 2 the data over a narrow range very close to the pure case, namely $0.95 \leqslant p \leqslant 0.999$. For reference purposes, the straight line in Fig. 2 has a slope of $-0.21$ and corresponds to the well-established persistence exponent for the pure case $p=1.0$. It's clear from Fig. 2 that we have deviations from the pure case even when $p=0.999$. For all values of ferromagnetic bond concentrations $p \neq 1$ we have a finite fraction of spins which *never* flip. Furthermore, the blocking probability, $P(\infty)$, also appears to be highly sensitive to the value of $p$.

In order to examine the behavior of the blocking probability, we plot in Fig. 3 the values extracted for $P(\infty)$ from Figs. 1 and 2 against the bond concentration. Note that Fig. 3 shows the values of $P(\infty)$ for a wide range of $p: 0 \leqslant p \leqslant 1$, including some values which have not been displayed in the earlier figures for clarity. Once again, the symmetry of the plot about $p=0.5$ acts as a consistency check on the numerics. The plot itself appears to have an interesting nonmonotonic, double-humped feature. In our model the average fraction of frustrated plaquettes, $f_{plaq}$ is given by [17]

$$f_{plaq} = 4p(1-p)[p^2 + (1-p)^2] \tag{6}$$

and there is a zero-temperature spin glass transition at $p_c \approx 0.11$ [18]. We see from Fig. 3 that the peak in the blocking probability coincides with this value of $p_c$. Furthermore, $f_{plaq}(p_c \approx 0.11) \approx 0.31 < 1/2$, the maximum value of $f_{plaq}$.

As explained earlier, for a blocked system, we can study the residual persistence $r(t) = P(t) - P(\infty)$. After having extracted the blocking probability for each $p$, we calculate $r(t)$. However, there is an error involved in estimating $P(\infty)$. As a consequence, the error in $r(t)$ is much greater than that in the original persistence probability. In Fig. 4 we show log-log plots of $r(t)$ against $t$ for three selected values of $p=0.1$, 0.5, and 0.9. In each case, we see that the decay of the residual persistence is algebraic over the time interval concerned. However, because of the uncertainty in the blocking probability, there are not inconsiderable error bars attached to the resulting (residual) persistence exponents.

Our estimates for the persistence exponents, $\theta(p)$, are plotted against the bond concentrations in Fig. 5. It would appear that $0.8 \leqslant \theta(p) \leqslant 1.1$ when $0.1 \leqslant p \leqslant 0.9$. For reference, the exponent for the pure case is indicated by the arrow. Note that although our data are not influenced by finite-size effects, it is nevertheless a nontrivial matter to extract the residual persistence exponent for $0 < p < 0.1$ because of the sensitivity to the estimate of $P(\infty)$. As can be deduced from Fig. 2, the closer we are to the pure case, the more difficult it is to estimate the blocking probability.

## IV. CONCLUSION

To conclude, we have presented data for the random bond Ising models on a square lattice. Our results confirm the existence of blocking in the system regardless of the amount of disorder present. The results are consistent with the presence of positive fractions of spins which flip finitely and infinitely often, respectively. We have also investigated the blocking probability and find interesting nonmonotonic behavior as a function of the ferromagnetic bond concentration. The persistence exponent has been extracted and found to be $\approx 0.9$, independent of the bond concentration over the range $0.1 \leqslant p \leqslant 0.9$. Although we know that $\theta(p=1.0) = 0.209 \pm 0.002$ [5], an accurate extraction of the residual persistence exponent for $p$ close to 1 is highly sensitive to the (assumed) value for $P(\infty)$. As a consequence, the complete nature of $\theta(p)$ over the entire range of bond concentrations remains to be established.

## ACKNOWLEDGMENT

H.F. would like to thank the University of Derby for financial support while this work was carried out.

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